Industrial & Applied Mathematics
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Distinguished Lecturer Series

Stochastic Approximation and the Analysis of Proportional Fair-Sharing Algorithms in Communications Theory

You are invited to a presentation by

Prof. Harold Kushner

Brown University

Date: Friday 26th Sep 2008

Time: 1400—1500hrs

Venue: Building Q1-01, Mawson Lakes Campus

Abstract

Recursive stochastic algorithms that take the form of discrete-time stochastic dynamical systems are known as stochastic approximations, and have an enormous variety of applications. A brief description of the basic algorithms and techniques of proof via weak convergence methods will be given. The most useful approach has been to show that the algorithm can be interpolated into a continuous-time system, whose limit is an ODE. The asymptotic properties of this ODE are the asymptotic properties of the stochastic approximation algorithm.
The ideas will be illustrated via an application to an important problem in communications theory, that of the behaviour of the Proportional Fair-Sharing Algorithms, that are of wide use in scheduling the transmission of data in cellular networks when there are many competing users. The channel capacities for the various users are varying randomly due to the movement of the mobiles. There is a conflict between full use (by selecting the user with the highest current rate) and fairness (which entails consideration for users with poor throughput to date). The Proportional Fair Scheduler is designed to deal with such conflicts. We put the algorithms on a sure mathematical footing and analyze their behaviour.
The sample paths of the throughputs converge to the solution of an intuitively reasonable ordinary differential equation, akin to a mean flow. The ODE has a unique equilibrium and it is characterized as optimizing a concave utility function. The results depend on the fact that the ODE has a special form that arises in problems with certain types of competitive behaviour. There is a large set of such algorithms, each one corresponding to a concave utility function. Various extensions will also be discussed.

Professor Harold Kushner has worked on nearly all parts of stochastic systems and control. His eleven books and about 250 papers contained many seminal developments. These include stochastic stability (Markov and non-Markov), nonlinear filtering, distributed and delay systems, stochastic variational methods and the stochastic maximum principle, stochastic approximation and recursive algorithms, efficient numerical methods for Markov chain models, numerical methods for general continuous-time systems, singular stochastic control, stochastic networks, heavy traffic analysis of queueing/communications systems, wide band noise driven systems, problems with small noise effects, approximation methods, nearly optimal control and filtering for non-Markovian systems.
He received :
the IEEE Field Award in Control Systems;
the SIAM Idalia and W.T. Reid Prize for contributions to control and stochastic differential equations;
the Louis Levy award from the Franklin Institute; and
the Bellman Heritage Award from the Americal Automatic Control Council.